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МНОГОКРИТЕРИАЛЬНАЯ ОПТИМИЗАЦИЯ ПОРТФЕЛЯ ОБОРУДОВАНИЯ ДЛЯ ДОБЫЧИ УГЛЯ: БАЛАНС ЗАТРАТ И ВЫБРОСОВ УГЛЕРОДА

MULTI-OBJECTIVE OPTIMIZATION OF COAL MINING EQUIPMENT PORTFOLIOS: BALANCING COSTS AND CARBON EMISSIONS

Аннотация: Предложена двухкритериальная модель смешанно-целочисленного нелинейного программирования для горного оборудования. Алгоритм NSGA-II балансирует затраты и выбросы, формируя границу Парето для энергоперехода.

Abstract: This study proposes a bi-objective mixed-integer programming model for mining equipment. The NSGA-II algorithm balances costs and emissions, generating a Pareto frontier to guide sustainable energy transitions.

1. Introduction

The extraction of coal resources maintains a measurable function in stabilizing regional socio-economic frameworks and supplying base-load energy, specifically within established industrial basins such as Kuzbass [1]. Concurrently, the operational mechanics of traditional fossil-fuel-based extraction generate greenhouse gas emissions that contravene international decarbonization protocols and national ecological mandates [2]. This structural divergence establishes a direct conflict between regional economic retention and global climate compliance. To reconcile these parameters, the modernization of mining infrastructure, specifically the substitution of diesel combustion equipment with electrified or hybrid technologies, functions as a primary mechanism for emission reduction. However, this technological substitution transitions the ecological conflict into a capital allocation problem, imposing severe capital expenditure (CAPEX) constraints on mining enterprises.

The evaluation of mining equipment portfolios predominantly relies on single-objective linear or mixed-integer programming frameworks, wherein the singular mathematical objective is the minimization of the total cost of ownership. Within these traditional structures, ecological limitations are frequently collapsed into rigid constraints or entirely omitted, restricting the capacity of the model to generate a transparent trade-off boundary between financial efficiency and carbon emission reductions. The absence of a multi-objective analytical space structurally limits decision-making, leading to suboptimal capital allocation risks when mining operations encounter volatile carbon taxation trajectories or shifting regulatory subsidies [3].

To address this methodological deficiency, this study formulates a Multi-Objective Mixed-Integer Non-Linear Programming (MOMINLP) model designed to optimize coal mining equipment portfolios. The mathematical architecture simultaneously targets the minimization of lifecycle costs and the reduction of carbon emissions under rigid operational constraints. Due to the computational complexity induced by binary investment variables, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is deployed to construct the Pareto frontier. By isolating non-dominated portfolio configurations, the proposed framework provides a quantitative mechanism for mining enterprises and regional policymakers to evaluate the exact marginal cost of ecological compliance, transitioning strategic planning from heuristic estimation to algorithmic decision-making.

2. Mathematical Methodology

2.1. Problem setting and assumptions

We consider a stylized open-pit coal mine that runs through a sequence of production stages, J = {1, 2, …, N}: drilling, excavation, hauling and crushing. At every stage j ∈ J the operator chooses among M technological options, I = {1, 2, …, M}, ranging from conventional fossil-fuel machinery to advanced electrified or hybrid equipment.

Buying equipment and running it are seperate decisions, so the model uses two kinds of variables. The binary variable $y_{ij} \in \{0,1\}$ records the investment: it equals 1 when technology i is adopted for stage j, and 0 otherwise. The continuous variable $x_{ij} \geq 0$ measures how intensively that technology is used, expressed as processed tonnage or allocated operating hours for technology i at stage j.

The mathematical formulation is grounded on three structural assumptions:

First, the mine has a fixed production target. A predefined output demand D_j must be satisfied at each stage j, so energy supply cannot be interrupted and emmissions cannot be reduced by simply extracting less.

Second, technologies cannot be bought in parts. Capital expenditures aries as a lump sum, which calls for an interger constraint on y_{ij} and rules out purchasing a fraction of a machine

Third, operating cost and emissions do not grow linearly with the intensity x_{ij}. Instead of assuming constant marginal costs, we ađ a quadratic term that captures the extra maintenace burden and the loss of efficiency when equipment works under heavy load.

2.2. The MOMINLP formulation

The mathematical architecture of the model is structured to quantify the trade-off between capital efficiency and ecological footprint. The first objective function, f_1, minimizes the total lifecycle cost of the equipment portfolio. This cost aggregates the initial Capital Expenditure (CAPEX) triggered by the strategic procurement decisions and the Operating Expenditure (OPEX) driven by tactical deployment. To reflect efficiency degradation under high-load conditions, the OPEX includes both a linear base cost and a quadratic penalty term. The financial objective is formulated as follows:

$$f_1 = \sum_{j\in J}\sum_{i\in I} C_{ij}^{cap} y_{ij} + \sum_{j\in J}\sum_{i\in I}\left(C_{ij}^{op} x_{ij} + P_{ij}^{op} x_{ij}^2\right) \quad (1)$$

where C_{ij}^{cap} denotes the discrete capital cost for deploying technology architecture i at stage j, C_{ij}^{op} represents the baseline marginal operating cost, and $P_{ij}^{op} > 0$ defines the quadratic degradation penalty scalar.

The second objective function, f_2, targets the minimization of aggregate greenhouse gas emissions. The same holds for emissions. Heavy machinery burns fuel less efficiently as it approaches peak load, and the efficiency drops exponentially, so emissions per unit of output rise non-linearly with intensity. The ecological objective is defined as:

$$f_2 = \sum_{j\in J}\sum_{i\in I}\left(E_{ij}^{base} x_{ij} + E_{ij}^{deg} x_{ij}^2\right) \quad (2)$$

where E_{ij}^{base} is the baseline emission factor and $E_{ij}^{deg} > 0$ captures the emission surge coefficient under suboptimal load conditions.

The optimization of these competing objectives is subjected to a system of structural constraints ensuring physical feasibility and economic limits:

$$\sum_{i \in I} x_{ij} \geq D_j, \quad \forall j \in J \tag{3}$$

$$\sum_{j \in J} \sum_{i \in I} C_{ij}^{cap} y_{ij} \leq B \tag{4}$$

$$x_{ij} \leq \Omega y_{ij}, \quad \forall i \in I, \forall j \in J \tag{5}$$

$$y_{ij} \in \{0,1\}, \quad x_{ij} \geq 0, \quad \forall i \in I, \forall j \in J \tag{6}$$

Constraint (3) enforces the production mandate, guaranteeing that the combined operational intensity of all selected technologies at stage j satisfies the required output demand D_j. Constraint (4) imposes a rigid financial boundary, restricting the total strategic CAPEX to the available budget B. Constraint (5) establishes the logical linkage between strategic and tactical variables via a strictly large positive constant Ω (Big-M formulation); it ensures that the continuous operational variable x_{ij} can only assume a positive value if the corresponding binary investment variable y_{ij} is activated ($y_{ij} = 1$). Finally, constraints (6) define the standard binary and non-negativity domains for the decision variables.

2.3. Solution approach via NSGA-II

The MOMINLP formulation presented in equations (1)-(6) is characterized by discrete non-convexity induced by the binary investment variables and structural non-linearities in the operational expenditure and emission functions. Traditional a priori scalarization techniques, such as the weighted-sum method, are mathematically restricted in this environment; they cannot reliably identify optimal solutions located in the concave regions of a non-convex Pareto frontier.

Consequently, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is deployed to solve the optimization problem. As an a posteriori meta-heuristic, NSGA-II circumvents the structural limitations of linear relaxation and gradient-based derivatives. It utilizes a fast non-dominated sorting mechanism to evaluate the fitness of discrete technological portfolios simultaneously across both the financial and ecological objectives. Furthermore, the algorithmic integration of a crowding distance metric preserves solution diversity during the crossover and mutation phases. This mechanism prevents premature convergence to local optima, ensuring the generation of a uniformly distributed Pareto optimal set that accurately reflects the trade-off boundary for subsequent economic analysis.

3. Numerical Experiment and Results

3.1. Data and algorithm settings

We test the MOMINLP framework on a stylized numerical experiment of a multi-stage open-pit coal mine. The process is split into three stages ($J = 3$): excavation ($j = 1$), hauling ($j = 2$), and crushing ($j = 3$). At each stage the mine can choose among four technologies ($I = 4$): legacy diesel machinery ($i = 1$), modern fuel-optimized diesel ($i = 2$), diesel-electric hybrid systems ($i = 3$), and fully electrified architectures ($i = 4$).

All input parameters are given as normalized relative unit indices (Table 1). This way the structural trade-off can be studied on its own, without interference from currency fluctuations or the geological features of any particular site. The parameter matrix deliberately embeds a strict economic-ecological contradiction. Advanced electrified systems ($i = 4$) require exponentially higher initial capital (C^{cap}) and incur higher operational expenditures (C^{op}) due to regional industrial electricity tariffs and cyclic battery maintenance. Conversely, these systems yield significantly lower baseline marginal emissions (E^{base}). Furthermore, the emission factor ($E_{i=4}^{base}$) incorporates Scope 2 emissions derived from the regional power grid, acknowledging that electrified mining equipment is not strictly zero-emission unless powered entirely by renewable energy. This parameter structure forces a genuine mathematical conflict with the low-cost, high-emission legacy diesel equipment, ensuring the generation of a non-degenerate Pareto frontier.

Table 1. *Normalized parameter matrix for the stylized mining operation*

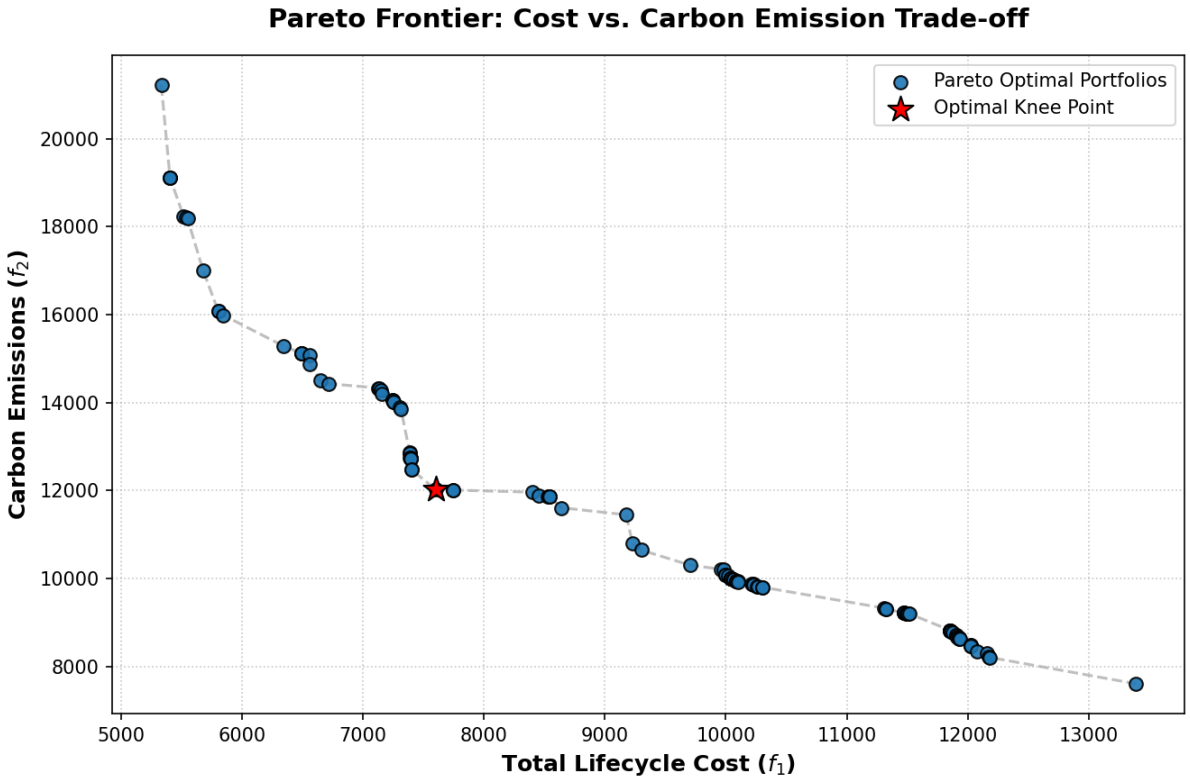
Stage (j)	**Technology Architecture (*i*)**	**C^{cap} (CAPEX)**	**$C^{(op)}$ (OPEX)**	**P^{op} (OPEX Penalty)**	**E^{base} (Emission)**	**E^{deg} (mission Penalty)**
1 (Excavation)	1 (Legacy Diesel)	100	5.0	0.01	20.0	0.080
	2 (Modern Diesel)	180	7.0	0.02	16.0	0.060
	3 (Hybrid)	350	9.0	0.04	10.0	0.030
	4 (Electrified)	600	12.0	0.06	3.0	0.010
2 (Hauling)	1 (Legacy Diesel)	150	8.0	0.02	25.0	0.100

	2 (Modern Diesel)	250	10.0	0.03	19.0	0.070
	3 (Hybrid)	450	14.0	0.05	12.0	0.040
	4 (Electrified)	750	18.0	0.08	4.0	0.010
3 (Crushing)	1 (Legacy Diesel)	80	4.0	0.01	15.0	0.050
	2 (Modern Diesel)	130	6.0	0.02	11.0	0.030
	3 (Hybrid)	220	8.0	0.03	6.0	0.020
	4 (Electrified)	380	10.0	0.05	1.0	0.005

The computational experiment is executed in Python using the pymoo algorithmic framework. Due to the discrete non-convexity of the solution space, the NSGA-II algorithm is configured with a population size of 100 individuals and evolved over 200 generations. To ensure comprehensive exploration of the Pareto frontier and prevent premature convergence, Simulated Binary Crossover (SBX) and Polynomial Mutation (PM) operators are applied.

3.2. Pareto frontier analysis and policy implications

Figure 1. *Pareto Frontier illustrating the trade-off between total lifecycle costs and carbon emissions*



The simulation generates a well-distributed Pareto frontier that maps the trade-off topology between lifecycle costs (f_1) and carbon emissions (f_2). Geometrically, the frontier exhibits a strict downward convexity, validating the theoretical premise that aggressive decarbonization mandates induce disproportionate escalations in capital expenditure.

The key quantity to read off the Pareto frontier is the “knee point”, where the curve bends. Up to this point, moving to modern diesel ($i = 2$) or hybrid architectures ($i = 3$) buys large emission cuts for a fairly small rise in total cost. Beyond it, as the solution approaches zero emissions limits, the hauling fleet has to be fully electrified ($i = 4$) and costs climb exponentially. In this asymptotic part of the curve the marginal abatement cost per unit of carbon keeps increasing geometrically, so each additional unit of capital brings less ecological benefit.

This nonlinear structure matters for regional planning. It suggests that strict environmental rules demanding immediate full electrification could push mining enterprises into serious financial difficulty. The Pareto distribution points to a better option: policymakers could set up counterpart funding (matching funds) or a carbon tax calibrated to the knee point. That would encourage the hybrid transition path while keeping the extraction sector economically viable.

4. Concluding remarks

In this paper we examined the conflict between financial efficiency and environmental compliance in coal mining and proposed a multi-objective

mixed-integer nonlinear programming (MOMINLP) model for it. Unlike standard single-objective cost minimization, the model puts a measurable price on the operational cost of decarbonization. NSGA-II handles the discrete, non-convex technology choices and returns a continuous Pareto frontier. From this frontier we can locate the inflection points where emission reductions are largest relative to the capital invested. For mining companies and regional policymakers, the framework replaces deterministic cost accounting with an algorithmic basis for allocating strategic capital during the transition, with economic and ecological objectives considered together.

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