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PROPAGATION OF WAVE PACKETS IN A COMPOSITE METALINE

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Composite R/LHTLs are a convenient model platform for realizing media with pronounced dispersion [1–3]. In works [4–6] it is shown that the propagation of intramodal wave packets in a planar optical waveguide based on a thin left-handed film can lead to new qualitative effects, for example, the effect of strong slowing of light at the carrier frequency coinciding with the frequency of zero group velocity of the mode [4,5]. In high-frequency R/LHTLs, just as in pure left-handed transmission lines (LHTLs), the propagation of band signals in the vicinity of cutoff frequencies, where the phase and group velocities, as well as the matching conditions, depend sharply on frequency, has been studied insufficiently. In the present work, an analysis and numerical simulation of wave packets in an R/LHTL loaded by an active resistance with an input voltage generator whose spectrum is pinned to one of the cutoff frequencies of the line is carried out.

For a lossless R/LHTL (see Fig. 1a), the telegrapher's equations with respect to the voltage $u(z,t)$ and current $i(z,t)$ in the time domain can be represented in the form:

$$-\frac{\partial u}{\partial z} = L^r \frac{\partial i}{\partial t} + \frac{1}{C^l}\int i dt\,,\ -\frac{\partial i}{\partial z} = C^r \frac{\partial u}{\partial t} + \frac{1}{L^l}\int u dt\,, \qquad (1)$$

where L^r and C^l – the inductance and capacitance of the series branch with dimensions H/m and F·m, respectively; C^r and L^l – the capacitance and inductance of the shunt branch with dimensions F/m and H·m, respectively.

Within the limits $L^l\to\infty$ and $C^l\to\infty$, these equations describe the RHTL (Fig. 1b), and under the conditions L^r=0 and C^r=0 they describe the LHTL (Fig. 1c).

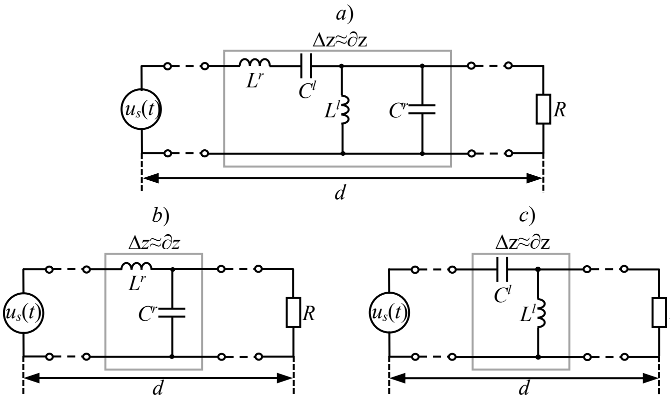

Fig. 1. R/LHTL (*a*), RHTL (*b*), and LHTL (*c*) with source $u_s(t)$ and load R, and equivalent circuits of their elementary sections.

Уравнения относительно спектральных плотностей напряжения $U(z,\omega)$ и тока $J(z,\omega)$ в частотной области могут быть получены в виде: The equations with respect to the spectral densities of voltage $U(z,\omega)$ and current $J(z,\omega)$ in the frequency domain can be obtained in the form:

$$\frac{dU(z,\omega)}{dz} = -Z(\omega)J(z,\omega),\ \frac{dJ(z,\omega)}{dz} = -Y(\omega)U(z,\omega), \tag{2}$$

where

$$Z(\omega) = i\rho_Z\left(\frac{\omega}{\omega_{cl}} - \frac{\omega_{cl}}{\omega}\right),\ Y(\omega) = i\frac{1}{\rho_Y}\left(\frac{\omega}{\omega_{cr}} - \frac{\omega_{cr}}{\omega}\right), \tag{3}$$

where $\omega_{cr}=(L^lC^r)^{-1/2}$, $\omega_{cl}=(L^rC^l)^{-1/2}$, $\rho_Z=(L^r/C^l)^{1/2}$, $\rho_Y=(L^l/C^r)^{1/2}$.

For the considered case, the solution of equations (2) can be obtained in the form:

$$U(\omega) = U_s(\omega)\left(\frac{\exp(-i\beta(\omega)z)}{1+\Gamma(\omega)\exp(-2i\beta(\omega)d)} + \frac{\Gamma(\omega)\exp(i\beta(\omega)z)}{\Gamma(\omega)+\exp(2i\beta(\omega)d)}\right), \tag{4}$$

where

$$\Gamma(\omega) = \left(R - R_\upsilon(\omega)\right)\left(R + R_\upsilon(\omega)\right)^{-1}, \tag{5}$$

$$R_\upsilon(\omega) = \sqrt{\rho_Z\rho_Y\,\omega_{cr}\left(\omega^2-\omega_{cl}^{\ 2}\right)/\omega_{cl}\left(\omega^2-\omega_{cr}^{\ 2}\right)}, \tag{6}$$

$$\beta(\omega) = s(\omega)\sqrt{(\rho_Z/\rho_Y)\left((\omega/\omega_{cl})-(\omega_{cl}/\omega)\right)\left((\omega/\omega_{cr})-(\omega_{cr}/\omega)\right)} \tag{7}$$

where $U_s(\omega)$ – is the spectral density of the source voltage $u_s(t)$, $s(\omega)$is chosen in accordance with condition [3]:

$$s(\omega) = \left[+1, \text{if } \omega > \max(\omega_{cr},\omega_{cl})\right] \vee \left[-1, \text{if } \omega < \min(\omega_{cr},\omega_{cl})\right].$$

The wave impedance of the R/LHTL does not depend on frequency and is equal to $R_\upsilon(\omega) \equiv R_\upsilon = (\rho_Z\rho_Y)^{1/2}$ only in the case of a balanced line, when $L^rC^l=L^lC^r$ ($\omega_{cl}=\omega_{cr}$). The frequency dependence $\beta(\omega)$ for balanced and unbalanced R/LHTLs is shown in Figs. 2*a* and 2*b*, respectively. For the unbalanced case, a stopband arises at $\omega_{cl}<\omega<\omega_{cr}$, where the constant $\beta(\omega)$ is imaginary. For the balanced case $L^rC^l=L^lC^r$ ($\omega_{cl}=\omega_{cr}=\omega_c$), the stopband disappears.

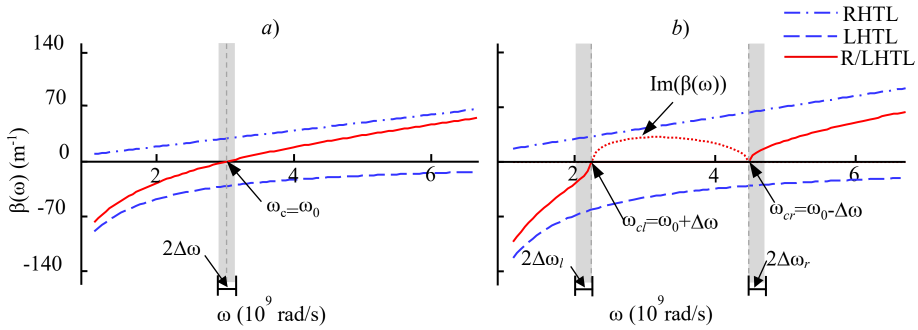

Fig. 2. Frequency dependence of the constant $\beta(\omega)$ in R/LHTL, RHTL, and LHTL for (*a*) balanced parameters (L^r=1 μH/m, L^l=1 nH·m, C^r=0.1 nF/m, C^l=0.1 pF·m) and (*b*) unbalanced parameters (L^r=2 μH/m, L^l=0.5 nH·m, C^r=0.1 nF/m, C^l=0.1 pF·m). The shaded regions show the qualitative position of the generator-signal spectrum.

Consider voltage wave packets in the line excited by a generator signal with a rectangular spectrum of width $2\Delta\omega$:

$$U_s(\omega) = \frac{\pi U_m}{i2\Delta\omega}\left[\Pi\left(\frac{\omega-\omega_0}{2\Delta\omega}\right) - \Pi\left(\frac{\omega+\omega_0}{2\Delta\omega}\right)\right], \tag{8}$$

where U_m – is the voltage amplitude, $\Pi(x)$ – is the rectangular function, and ω_0 – is the central frequency, the variants of whose position with respect to the dependence $\beta(\omega)$ shown in Fig. 2.

At $R=R_\upsilon(\omega_0)$, the voltage in the line can be obtained in the form:

$$u(z,t)=\frac{U_m}{4\Delta\omega}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}\mathrm{Im}\left[\frac{\exp\left(i\left(\omega t-\beta(\omega)z\right)\right)}{1+\Gamma(\omega)\exp(-2i\beta(\omega)d)}+\right.\\\left.+\frac{\Gamma(\omega)\left(\exp\left(i\left(\omega t+\beta(\omega)z\right)\right)\right)}{\Gamma(\omega)+\exp(2i\beta(\omega)d)}\right]d\omega, \qquad (9)$$

For pure RHTL/LHTL and for the balanced R/LHTL ($R_\upsilon(\omega)\equiv R$ and $\Gamma(\omega)\equiv 0$), the characteristic impedance does not depend on frequency. At $\omega_c=\omega_0$, expression (9) for the case of RHTL and LHTL can be reduced to the form:

$$u(z,t)=U_m\,\mathrm{sinc}\left\{\left[t-\left(|\beta_c|/\omega_c\right)z\right]\Delta\omega\right\}\sin\left(\omega_c t-\beta_c z\right), \qquad (10)$$

where $\beta_c=\omega_c(L^rC^r)^{1/2}$ for the RHTL and $\beta_c=-\omega_c^{-1}(L^lC^l)^{-1/2}$ for the LHTL. For the balanced R/LHTL, the expression for the voltage $u(z,t)$ is also reduced to a simple form:

$$u(z,t)=U_m\,\mathrm{sinc}\left(\left(t-2\sqrt{L^rC^r}\,z\right)\Delta\omega\right)\sin\left(\omega_c t\right), \qquad (11)$$

Here the “sinc” is type envelope propagates with the group velocity. In a balanced R/LHTL this velocity is two times smaller than in the RHTL and LHTL, and the carrier has zero phase velocity. These features are due to the fact that in one half-band $\omega\in[\omega_c-\Delta\omega,\omega_c)$ the packet is formed by harmonic waves with negative phase velocity, and in the other half-band $\omega\in(\omega_c,\omega_c+\Delta\omega]$, by harmonic waves with positive phase velocity.

In Fig. 3*a* and 3*b*, the space-time evolution of the envelope and the carrier, respectively, of the wave packet in a balanced line of length d=1 m is shown. The calculation of all curves is performed with the same parameters as in Fig. 2. The parameters common to all three lines are set to the following values: $\omega_0=\omega_c$, $R(\omega_c)$=100 Ω, U_m=1 V, $\Delta\omega=\omega_c/10$. In Fig. 3*a*, the envelopes in the RHTL and LHTL coincide; the envelope in all three lines moves from the source to the load. In Fig. 3*b*, the carriers in the RHTL and LHTL propagate in opposite directions, while in the balanced R/LHTL the carrier is stationary.

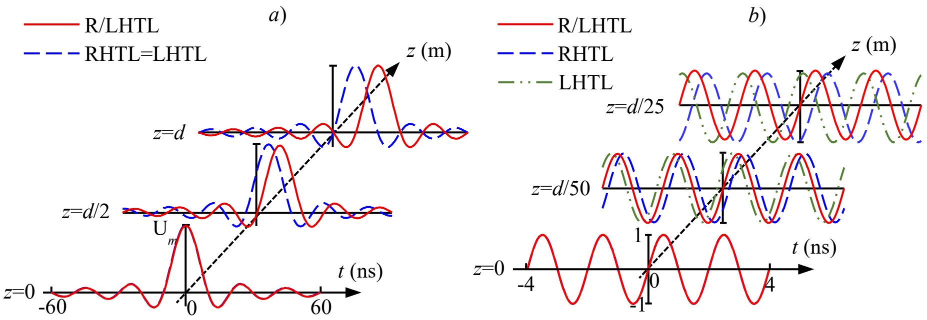

Fig. 3. Space-time evolution of: (*a*) envelopes and (*b*) carrier oscillations in the RHTL, LHTL, and balanced R/LHTL.

In unbalanced R/LHTL, the impedance $R_\upsilon(\omega)$ depends on frequency ($\Gamma(\omega)\neq 0$). Therefore, the voltage in the line is a superposition of forward- and backward-propagating (incident and reflected) wave packets. Below, a narrowband packet is considered for two positions of the spectrum: the lower boundary of the spectrum at the upper cutoff ($\omega_0=\omega_{cr}+\Delta\omega$), and the upper boundary at the lower cutoff ($\omega_0=\omega_{cl}-\Delta\omega$) (see Fig. 2*b*). Numerical simulation showed that in these cases the contribution of the reflected packet to the voltage, and it, can be neglected.

Within the framework of this approximation, the general relation for the voltage in the line can be obtained in the form of a superposition of two incident wave packets with quadrature carriers [7]:

$$u(z,t)\cong I(z,t)\cos(\omega_* t)+Q(z,t)\sin(\omega_* t), \tag{12}$$

where the carrier frequency, $\omega_*\neq\omega_0$, is taken not at the center of the source spectrum, but equal to the boundary frequency of the band ($\omega_*=\omega_{cr}$ or $\omega_*=\omega_{cl}$). At the same time, the carrier oscillation does not move in space (the phase velocity of the packets is equal to zero).

The results of numerical simulation of the slowly varying envelopes of the quadratures introduced into consideration (see (12)) are presented in Fig. 4 for the cases corresponding to Fig. 2*b*, when $\Delta\omega_r=\omega_{cr}/10$ and $\Delta\omega_l=\omega_{cl}/10$.

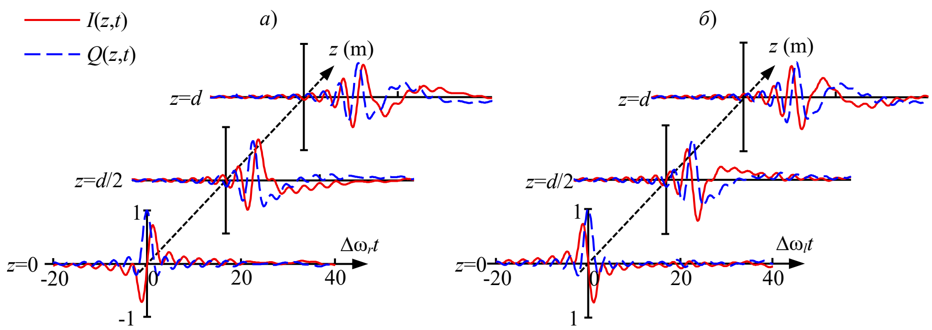

Fig. 4. Space-time dynamics of the quadrature envelopes of voltage wave packets propagating in the forward direction, in an unbalanced R/LHTL for the cases $\omega_0=\omega_{cr}+\Delta\omega$ and $\omega_0=\omega_{cl}-\Delta\omega$.

It follows from the figure that the shape of the quadrature envelopes changes noticeably along the line. These changes indicate an increase in that part of the packet energy which is concentrated in their temporal tails located after the main lobe. The cause of such a redistribution of energy is the frequency dependence of the coefficient $\Gamma(\omega)$, which arises owing to the strong dispersion of the wave impedance $R_\upsilon(\omega)$.

Thus, excitation of an unbalanced R/LHTL with a resistive load by a microwave-range voltage signal with a spectrum pinned to one of the cutoff frequencies leads to the appearance of incident and reflected voltage wave packets. For narrowband excitation, the contribution of the reflected packets to the voltage along the line can be neglected. Dispersion of the wave impedance of the line leads to noticeable distortions of two incident wave packets with quadrature carriers. The analytical description of these packets allows the choice of the cutoff frequency as the frequency of the carrier oscillation, which in this case does not propagate along the line. The absence of frequency dispersion of the wave impedance of a balanced R/LHTL makes it possible to realize a matched-load regime. Excitation of a balanced R/LHTL by a band packet with central frequency equal to the frequency of zero phase velocity leads to distortion-free propagation of a wave packet with a carrier oscillation not propagating along the line.

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